SEPARATION OF A VAPOR BUBBLE AND CALCULATION OF THE CRITICAL BUBBLE RADIUS

I. M. Fedotkin, S. M. Konstantinov, and A. A. Tereshchenko

UDC 536.423.1

On the basis of the solution to the equation of motion of a variable mass, a formula is derived for analytically calculating the critical radius of a vapor bubble in a boiling liquid during free flow of the latter.

The formula for the critical radius of a vapor bubble

$$R_0 = 0.01\theta \sqrt{\frac{\sigma}{g(\rho' - \rho'')}},$$

was proposed by W. Fritz [1] and subsequently refined by various other authors. S. G. Teletov [2] derived a similar formula for a sloping surface.

The equilibrium equation of forces, including the surface tension as well as the Archimedes force and the pressure force on a bubble surface, was solved in [3] with both the dynamic and the static components taken into consideration.

A similar summation of forces was used in [4-5].

Here the authors will try to determine the bubble radius before separation, when the buildup is complete and a "neck" is formed, taking into account other additional acting forces.

A vapor bubble separating from a surface will be represented by a material point of variable mass [6]. A material point is defined as a quantity of particles which at the instant of time t exist within a region bounded by some reference surface (in our case the interphase boundary) and assumed to be moving continuously together with some geometrical point, this point being the center of the bubble. The shape of a bubble is, moreover, assumed spherical. From analytical mechanics of variable-mass bodies we have the well known equation of motion

$$m\frac{d\mathbf{v}}{dt} = \mathbf{F} + \mathbf{B}. \tag{1}$$

The vector of the reaction force is

$$\mathbf{B} = \mathbf{J} - \Sigma m\,\mathbf{w}_p + \mathbf{\Phi}. \tag{2}$$

The Coriolis force is $J = 0$ here, because the motion of the material point is translatory only. The acceleration of particles $\mathbf{w}_p$ contained within a material point is zero relative to the auxiliary system of coordinates. The origin of the latter coordinate system is located at the center of the given spherical material point. With all these stipulations we have

$$\mathbf{B} = \mathbf{\Phi}. \tag{3}$$

The magnitude of the impulse (impact) force produced as a result of new particles joining the material point is

$$\Phi = -\Sigma\frac{d\mu}{dt}\Delta u. \tag{4}$$

Engineering Institute of the Food Industry, Kiev. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 5, pp. 831-835, May, 1973. Original article submitted August 7, 1972.

TABLE 1. Comparison between the Critical Radius Calculated According to Formula (18) and Measured in Tests

P, bars	R_{0exper}, mm [11-13]	R_{0exper}, mm [14] pure surface	R_{0exper}, mm [14] extra treated surface	R_{0theor}
0.98	—	—	1.24	1.59
1.01	1.0—1.25	—	—	—
1.33	0.75	0.73	—	—
3.14	0.4	0.7	—	—
3.20	0.35	—	0.355	0.32
10.0	0.15	—	—	—
11.6	—	0.28	—	—
11.8	—	—	0.215	0.1
47.7	—	—	0.145	0.023
77.5	—	—	0.130	0.014
95.7	—	—	0.085	0.01

When a vapor bubble separates from a horizontal surface, then the principal vector of external forces F includes the pulling force F_1 and the resistance force of the liquid F_2. Henceforth one may abandon the vector notation and regard just the absolute values of quantities with the appropriate algebraic signs, because in this particular case all forces are collinear in the same or in opposite directions.

Some authors suggest that surface tension be regarded as the external force, namely

$$F_3 = 2\pi R_0 f\theta\sigma.$$

Here we will disregard this force for the following reasons:

first of all, the "neck" of a vapor bubble ruptures at the instant of separation and, consequently, the circular area on which surface tension is acting contracts to a point;

secondly, the force is determined by both the magnitude of surface tension and by some function of the wetting angle θ. The magnitude of surface tension is defined either under static conditions or during slow separation [7]. Test data obtained by photographic and cinematographic recording indicate the existence of a "neck" during separation. After separation, the bottom part of this "neck" remains at the heater surface. Consequently, a determination of the wetting angle is not relevant here in this sense, the latter being defined at the contact of liquid and vapor with a solid surface. Here, at the instant of separation we measure a certain angle between the generatrix of the vapor surface and the plane of the heater surface at the point of separation.

Taking all this into consideration, we write the general equation for the dynamic balance of forces acting during the separation of a bubble from a horizontal heater surface when the liquid boils but does not move relative to that heater surface:

$$m\frac{dv}{dt} = F_1 + F_2 + \Phi. \tag{5}$$

Let us now analyze the quantities in expression (5) as they apply to a vapor bubble. The mass m of a vapor bubble includes that of vapor proper and that of adjoining liquid [8]:

$$m = m_1 + m_2, \tag{6}$$

where $m_1 = \rho''V$ and $m_2 = C\rho'V$.

The pulling force is

$$F_1 = Vg(\rho' - \rho''). \tag{7}$$

The force of resistance to bubble motion will be determined as follows. On the basis of test data pertaining to the critical size of a bubble and on the basis of theoretical calculations pertaining to the velocity of the interphase boundary during bubble buildup, we find that the Reynolds number is of the order of $Re \approx 10^4$. At the instant of separation a bubble is assumed to be spherical, and the resistance coefficient for a sphere is $\zeta = 0.42$ so that the resistance force becomes [9]

$$F_2 = \zeta S\rho' \frac{v^2}{2}. \tag{8}$$

The impulse (impact) force can be determined as follows.

During passage through the interphase boundary, the velocity of particles changes stepwise [10]:

$$\Delta u = \frac{g_p}{\rho''}\left(1 - \frac{\rho''}{\rho'}\right). \tag{9}$$

Considering that $g_p = g/r$, we can write

$$g_p = \rho'' \dot{R}. \tag{10}$$

The rate of particles piercing through the boundary is characterized by the change of bubble mass, i. e.,

$$\frac{d\mu}{dt} = \rho'' \frac{dV}{dt}. \tag{11}$$

Taking all this into account, we write the expression for the impulse force as follows:

$$\Phi = \rho'' \dot{R}\left(1 - \frac{\rho''}{\rho'}\right)\frac{dV}{dt}. \tag{12}$$

In the general Eq. (5) we express all quantities explicitly and obtain

$$V(\rho'' + C\rho')\frac{dv}{dt} = Vg(\rho' - \rho'') - \zeta S\rho' \frac{v^2}{2} + \rho'' \dot{R}\left(1 - \frac{\rho''}{\rho'}\right)\frac{dV}{dt}. \tag{13}$$

As the condition for bubble separation from a heater surface we let

$$v = \dot{R}. \tag{14}$$

Indeed, the velocity of the bubble center v after separation must be higher than the velocity of the bubble wall $\dot{R}$, lest the bottom of the sphere keep adjoining the heater surface. At the instant when both these velocities become equal, as velocity v increases relative to $\dot{R}$, will be the instant of bubble separation.

In order to determine the velocity of the bubble wall, we use the model of bubble buildup at a heater surface according to [11]. The choice of this model is dictated by the fact that most heat at the instant of separation is supplied to the bottom "adhering" region of the superheated liquid layer on the heater surface. The velocity of the bubble wall, i. e., the rate of bubble buildup is

$$\dot{R} = \beta \frac{\lambda\vartheta}{r\rho''}\frac{1}{R}. \tag{15}$$

When $\theta = 0$, then $\beta = 6.9 \approx 7.0$ [10]. For this buildup model, under the conditions stipulated here, we have

$$\frac{dv}{dt} = \ddot{R} = -\left(\beta \frac{\lambda\vartheta}{r\rho''}\right)^2 \frac{1}{R^3}. \tag{16}$$

Solving Eq. (13) with the separation condition taken into account, we obtain the following formula for the critical radius:

$$R_0 = \left(7.0 \frac{\lambda\vartheta}{r\rho''}\right)^{2/3}\left(\frac{C}{g} + \frac{0.16}{g} - \frac{3}{g}\frac{\rho''}{\rho'}\right)^{1/3}, \tag{17}$$

or for $\rho' \gg \rho''$

$$R_0 = \left(7.0 \frac{\lambda\vartheta}{r\rho''}\right)^{2/3}\left(\frac{C}{g} + \frac{0.16}{g}\right)^{1/3}. \tag{17a}$$

Coefficient C = 1.75 when angle $\theta = 0$. Formulas (17) and (17a) become then

$$R_0 = \left(7.0 \frac{\lambda\vartheta}{r\rho''}\right)^{2/3}\left(\frac{1.91 - 3\frac{\rho''}{\rho'}}{g}\right)^{1/3}, \tag{18}$$

or for $\rho' \gg \rho''$

$$R_0 = 2.12\left(\frac{\lambda\vartheta}{r\rho''}\right)^{2/3}. \tag{18a}$$

It is evident from (18) that, as the pressure of the boiling liquid rises, the critical radius of bubbles will decrease — which agrees with available test data.

The theoretical data are compared with test data [12-15] in Table 1. Only in [15] have we found superheat values for a heater surface and, therefore, the theoretical values of the critical radius have been given here for pressure stipulated in [15]. It is noteworthy that the agreement between theoretical and test values is closest for pressures up to 10 bars.

NOTATION

R_0 is the critical radius of a vapor bubble, m;
σ is the surface tension, N/m;
θ is the wetting angle, in ang. deg.;
$f(\theta)$ is the function of angle θ;
g is the acceleration of gravity, m/sec^2;
ρ' is the density of liquid, kg/m^3;
ρ'' is the density of vapor, kg/m^3;
t is the time, sec;
m is the mass of a material point, kg;
v is the velocity vector of the center of a material point, m/sec;
$\mathbf{F}$ is the principal vector of external forces, N;
$\mathbf{B}$ is the vector of the reaction force, N;
$\mathbf{\Phi}$ is the vector of the impulse (impact) force, N;
$d\mu/dt$ is the rate of particles passage in a given direction, kg/sec;
Δu is the step change in the velocity of particles passing at a given instant of time, m/sec;
V is the volume of a vapor bubble, m^3;
C is the coefficient of adjoining mass;
ξ is the coefficient of hydraulic resistance;
S is the median section area of a bubble, m^2;
g_p is the rate of phase transformation, $kg/m^2 \cdot sec$;
q is the thermal flux density needed for changing the aggregate state of the substance at the interphase boundary, per unit area of interphase boundary, W/m^2;
r is the latent heat of evaporation, J/kg;
$\dot{R}$ is the velocity of the interphase boundary, m/sec;
β is a coefficient;
λ is the thermal conductivity, $W/m \cdot °C$;
ϑ is the temperature difference between a superheated wall and the liquid away from it, °C.

LITERATURE CITED

1. W. Fritz, Phys. Zeitschrift [German], No. 11, 379 (1935).
2. S. G. Teletov, Izv. ÉNIN Akad. Nauk SSSR, 11 (1940).
3. V. M. Borishanskii and K. A. Zhokhov, Inzh. Fiz. Zh., 15, No. 4 (1968).
4. A. A. Voloshkov and A. V. Vurgaft, Inzh. Fiz. Zh., 19, No. 1 (1970).
5. V. F. Prisnyakov, Inzh. Fiz. Zh., 19, No. 5 (1970).
6. V. S. Novoselov, Analytical Mechanics of Variable-Mass Systems [in Russian], Izd. Leningradsk. Univ. (1969).
7. N. K. Adam, Physics and Chemistry of Surfaces [Russian translation], Gostekhizdat, Moscow —Leningrad (1947).
8. L. Milne-Thomson, Theoretical Hydrodynamics, Macmillan, New York (1968).
9. S. S. Kutateladze and N. V. Borishanskii, Textbook on Heat Transmission [in Russian], Izd. GÉI, Moscow—Leningrad (1959).
10. S. S. Kutateladze, Fundamentals of Heat Transfer Theory [in Russian], Izd. Nauka, Novosibirsk (1970).
11. D. A. Labuntsov, Izv. Akad. Nauk SSSR Otdel. Tekh. Nauk, Énergetika i Transport, No. 1 (1963).
12. N. N. Mamontova, Prikl. Mekhan. i Tekh. Fiz., No. 3 (1963).
13. V. I. Tolubinskii and N. Yu. Ostrovskii, in: Convective Heat Transfer [in Russian] (V. I. Tolubinskii, editor), Izd. Naukova Dumka, Kiev (1965).
14. V. I. Tolubinskii and N. Yu. Ostrovskii, Internatl. J. Heat and Mass Transfer, 9, No. 12 (1966).
15. D. A. Labuntsov, B. A. Kol'chugin, V. S. Golovin, É. A. Zakharova, and L. N. Vladimirova, in: Heat Transfer in Power Plant Components [in Russian], Izd. Nauka, Moscow (1966).